A filter banks design using multiobjective genetic algorithm for an image coding scheme

A. Boukhobza[*], A. Taleb Ahmed[*], A. Bounoua[**], N. Taleb[**]

** Laboratoire de recherche LAMIH UMR CNRS, UHVC*
Le Mont Houy, 59313 Valenciennes cedex 9

*** Laboratoire de recherche RCAM, Université de Sidi Bel Abbes, Avenue Larbi-Ben-M'hidi, BP 89, Sidi bel abbes 22000, ALGERIE*

aek_boukhobza@yahoo.ca, abdelmalik.taleb-ahmed@univ-valenciennes.fr, nacerbounoua@yahoo.fr, ne_taleb@univ-sba.dz

Sections de rattachement : 61
Secteur : Secondaire

RÉSUMÉ. In this paper, we present a global optimisation method based on a multi-objective Genetic Algorithm (GA) for the design of filter banks in a lossy image coding scheme. To be effective, the filter banks should satisfy a number of desirable criteria related to such scheme. We formulate the optimization problem as multi-objective and we use the Non-dominated Sorting Genetic Algorithm approach (NSGAII) to solve this problem by searching solutions that achieve the best compromise between the different objectives criteria, these solutions are known as Pareto Optimal Solutions. Flexibility in the design is introduced by relaxing Perfect Reconstruction (PR) condition and defining a PR violation measure as an objective criterion to maintain near perfect reconstruction (N-PR) solutions. Furthermore, the optimized filter banks are near-orthogonal. This can only be made possible by minimizing the deviation from the orthogonality in the optimization process. Our designed filter banks lead to a significant improvement in performance of coding with respect to the 9/7 filter bank of JPEG2000 at high compression ratios and offer a slight improvement at low compression ratios.

MOTS-CLÉS : filter banks design, image coding, multi-objective optimization, genetic algorithms.

1. Introduction

In a biorthogonal filter bank, Perfect Reconstruction (PR) linear phase system can be constructed by imposing symmetric filters. In addition to the PR and phase linearity of the system, we should consider the following criteria in order to design effective filter banks for image coding (Vitterli et Ko., 1995), namely: Energy compaction capability or coding gain, frequency selectivity, orthogonality, and Regularity.

In this work, we present global optimization design of biorthogonal filter banks for lossy image coding. In such scheme, we can tolerate some errors and maintain N-PR filter banks by defining a PR violation as an objective function to be minimised. The relax of the PR condition introduces flexibility in the design by allowing the investigation of the unexplored regions of search space where better performing filter banks can be obtained. Moreover, we design a near orthogonal filter bank by assigning the measure of deviation from orthogonality as an objective. This is very helpful, especially, where progressive transmission is required.

Practically, it is not possible to design a filter bank that satisfy maximally all the above criteria. A multi-objective genetic approach called NSGAII (Deb 1999) is used to find Pareto optimal solutions that make all possible tradeoffs among competing objectives through evolution.

This article is organized as follows. The new criteria used in this paper for the design of filter banks are defined in section 2. A multi-objective GA for designing filter banks for image coding purpose is presented in Section 3. In section 4 we evaluate the performances of compression of the optimized filter banks for a set of test images. Finally, Section 5 concludes the paper with a summary of our work.

2. Filter bank design criteria

In the following sub-sections we focus only on some measure functions introduced in this work as design criteria to construct effective filter bank for image compression. The other criteria are frequently used in the design of filter banks and can be found in (Shang et Li, 1999).

2.1 Perfect reconstruction violation measure

In subbands coding, a transformation is achieved by using a filter bank. Unlike the lossless coding scheme, at lossy coding case this constraint is not strictly imposed and small distortion can be tolerable.

In our work, we are fixed to the design of symmetric odd length FIR filters. In this case, PR filter banks satisfy the following conditions (Horng et Wi., 1992):

1. Impose $G_0(z)=H_1(-z), G_1(z)=-H_0(-z)$ in order to remove the aliasing (e.g. fig.1).

2. The total length of them is a multiple of 4, $N_0 + N_1=4m$, Where N_0 and N_1 are the filters lengths of h_0 and h_1 respectively.

3. Satisfy the constraint equations :

$$\theta\left(i-\frac{N_0+N_1}{4}\right)=\sum_{k=1}^{2i}(-1)^{k-1}h_0(2i+1-k)h_1(k) \tag{1}$$

And $i=1,2,\ldots,(N_0+N_1)/4$

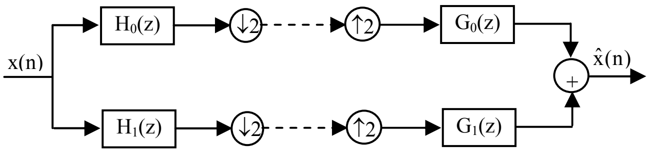

Figure 1. *Biorthogonal filter bank*

Where $h_0(n)$, n=1, ... , N_0, are coefficients of $H_0(z)$; $h_l(n)$, n=1, ..., N_1, are coefficients of $H_1(z)$; and $\theta(x) = 1$ if $x = 0$, and 0 otherwise.

The PR condition reduces the number of free parameters that can be used in the filter bank optimization. This allows us for searching filter banks that improve the coding performance by searching in larger feasible solution space.

In this work, we don't impose the PR constraint, and we use a measure function to quantify the PR violation (PRV) of filter banks. The PRV measure function is expressed as:

$$PRV=\sum_{i=1}^{(N_0+N_1)/4}\left(\theta\left(i-\frac{N_0+N_1}{4}\right)-\sum_{k=1}^{2i}(-1)^{k-1}h_0(2i+1-k)h_1(k)\right)^2 \tag{2}$$

This function is used in experimentation to limit the search process for near PR filter banks.

2.2 Deviation from the orthogonality

Orthogonal transforms are energy preserving. Therefore, the distortion in the transformed image after quantization is the same in reconstructed image. In the biorthogonal case, this definition does not hold directly and a weight for each subband is required which takes into consideration the different energy contributions from different subbands (Vitterli et Ko., 1995).

Orthogonal (unitary) filter banks satisfy the power complementary property:

$$\sum_{k} \left| H_k(e^{j\omega}) \right|^2 = 1, \; k = 0,1 \tag{3}$$

So, to design near orthogonal filter banks, we define an adequate measure function by:

$$DO = \int_0^{\pi} \left(1 - \left(\left| H_0(e^{j\omega}) \right|^2 + \left| H_1(e^{j\omega}) \right|^2 \right) \right)^2 d\omega \tag{4}$$

3 A multiobjective Genetic algorithm for the design of filter banks

A general multi-objective optimization problem consists of a number of objectives to be optimized simultaneously and is associated with a number of inequality and equality constraints. Such a problem can be stated as follows:

$$\begin{aligned} &\text{Minimize (or Maximize)} \;\; f_i(x) \qquad i = 1,\ldots,N \\ &\text{subject to :} \begin{cases} g_j(x) = 0, j = 1,\ldots,M \\ h_k(x) \le 0, k = 1,\ldots,K \end{cases} \end{aligned} \tag{5}$$

The f_i are the objective functions, N is the number of objectives, x is a vector whose p components are the design or decision variables, g_i and h_i are the constraint functions. Generally, the objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can degrade results with respect to the other objectives. Generally, it is difficult to combine the above objectives both to formulate a single objective function. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution. Such a set is referred to as the *Pareto optimal solution set*.

In a minimization problem, for N objective functions, a feasible solution **X** is dominated by feasible solution **Y** if:

$$\begin{aligned} &\forall i = 1,\ldots,N, \quad f_i(X) \ge f_i(Y) \quad \text{and} \\ &\exists j = 1,\ldots,N, \quad f_j(X) > f_j(Y) \end{aligned} \tag{6}$$

A solution is said to be Pareto optimal if it is not dominated by any other solution in the solution space. Furthermore, that solution is said to be *nondominated solution*. The set of all feasible nondominated solutions are called the *Pareto optimal set*. The corresponding objective function in the objective space constitutes a *Pareto front*.

Practically, we search filters h_0 and h_1 that maximise the coding gain and minimise the three individual objective functions, namely, the energies of the filter approximation error, the PR violation measure function, and the deviation from the orthogonality measure. To confine the search process in feasible solution space, we impose a limit to the PR violation measure function as a constraint and we move it from the above objective functions. We have to determine only $(N_0 + N_1)/2 - 3$ coefficients since the remaining coefficients are obtained, firstly, by imposing smoothness constraints and regularity of order two (Shang et Li, 1999), and secondly, by applying symmetry.

Our multi-objective optimization problem is formulated as follows:

$$\min_{h_0,h_1}(Objf_1, Objf_2, Objf_3), \text{ and } \begin{cases} Objf_1 = E_{ps} \\ Objf_2 = \dfrac{1}{CG^2} \\ Objf_3 = DO \end{cases} \qquad (7)$$

Subject to the constraint:

$$PRV < \alpha$$

Where:

- E_{ps}: is the sum of stopband energy and passband energy of a filter approximation error in h_0 and h_1 (Shang, et Li, 1999) used to quantify the frequency selectivity of the filter banks.

- CG: is the filter dependent coding gain of (Katto et Ya., 1991) calculated for 5 level tree structured filter bank. It is used to measure the energy compaction of filter banks.

- α is a small value (e.g., of order 10^{-5}) used here to maintain NPR filter banks in the evolutionary search process.

In our case, A set of coefficients of the analysis filter bank (h_0, h_1) are treated as chromosomes which are optimized by the NSGA II (Deb 2002) approach to obtain a set of filter banks that minimise the prescribed objective functions with a satisfactory level.

The particularity of NSGAII is that in addition to the Pareto nondomination principle used in the conventional multi-objective genetic algorithm: a crowding operator is used to maintain diversity in the population and an elitism mechanism is introduced to prevent the loss of better solutions once they are found during the genetic evolution (Deb 2002).

In our problem formulation, the presence of constraint engenders two types of solution: feasible solutions and infeasible solutions. For handling this constraint, two different processing are performed for each type of solution in the non-domination sorting stage. Firstly, feasible solutions are sorted using the crowded-comparison operator (Deb 2002) and using the three objectives values and the constraint value. Secondly, infeasible solutions are sorted based on the constraint violation values. So, any feasible solution is considered better than any infeasible solution and among two infeasible solutions, the solution with the smaller constraint violation is the best.

4 Experimental results

Before evaluation of our results, we give some important parameters used in our simulation work. In the evaluation of the frequency selectivity of filter banks, we set the pass-band edge to 1 and stop-band edge to π-1 for calculating the pass-band and the stop band non-desired energies. In our genetic algorithm, real valued filter coefficients are used for chromosome construction to avoid very long strings. A simulated binary crossover operator with a distribution index of 20 and probability of 0.9 is used (Raghuwanshi et Ka., 2004). Also, a polynomial mutation of distribution index 20 and probability of 0.01 is applied (Raghuwanshi et Ka., 2004). The population size is set to 100, in order to find as many Pareto optimal solutions as possible. Chromosomes of the initial population are obtained by randomly generating filter coefficients between [-1,1]. To enforce the N-PR system we set the constraint α to 10^{-5}. The maximum generation is set to 2000 generations.

The fundamental difficulty in testing the image compression system is how to decide which test images to use for the evaluations. In our work, it is very interesting to design filter banks that can perform well for any test image. Therefore, a series of images which have different frequency contents has been selected for evaluation of filter banks coding performance. Their spatially characteristics are evaluated using the Spatial Frequency Measure (Eskicioglu, et Fi., 1995). All are 512×512 grayscale images.

In experimentation, the SPIHT codec (Said et Pe., 1996) is employed for evaluation of the performance of the optimized filter banks. Five levels of wavelet decomposition have been employed. In what follows, we use the notation N_0/N_1 to indicate that the filter bank has low-pass and high-pass analysis filters of lengths N_0 and N_1 respectively.

In table 1, we provide the performances in $PSNR_{dB}$ (Peak Signal to Noise Ratio)/compression ratio of the three optimized filter banks 9/7 (FB9/7), 13/11 (FB13/11), and 17/11 (FB17/11) for an example of three test images. Since the filter banks are optimized for lossy coding purposes, we consider only moderate and high compression ratios. To qualify the effectiveness of our design method, we compare the coding performances of our optimized filter banks with the performances of the 9/7 biorthogonal filter bank of JPEG2000, which is used as reference and it is labelled by "FB_{REF}9/7" (Villasenor, Be., et Li., 1995). So, we introduce in the same table the performances of this filter bank where the best result is highlighted in each case.

For these three images, our optimized filter banks provide the best PSNRs for all compression ratios. We remark that longer filter banks resulted in a further increase in PSNR with respect to the 9/7 reference filter bank. Statistically, over the three test images, our optimal design outperforms $FB_{REF}9/7$ filter bank at least 84.37% of the time. We have obtained in average an improvement of 0,18dB for the FB13/11 and FB17/11, and 0,1dB for the FB9/7. Our method leads to an improvement of performances sometimes more than 0.5dB.

Figure 2 shows a 3D scatter plot of a set of Pareto optimal solutions obtained for the 9/7 filter bank. At the expense of PR condition violation, the optimized filter banks have better characteristics (objective functions) than those of the filter $FB_{REF}9/7$ which justifies theirs better performances especially for high compression ratios.

In this work, a method based on genetic algorithm was presented for the optimization of filter banks for a lossy image coding scheme. The problem of optimization is to find a set of filter bank coefficients that satisfy multiple objectives which are used to measure the effectiveness of filter banks in such scheme. The problem was formulated as multi-objective and solved using the NSGAII algorithm. A special constraint was also introduced to identify infeasible solutions thereby confining the search for the N-PR filter banks. From simulation results, it is shown that our optimized filter banks outperform the 9/7 filter bank of JPEG2000 for the majority of tested cases.

Particularly, we have obtained near orthogonal filter banks. This objective is achieved by the minimization of the deviation from the orthogonality in the optimization process. This is very helpful for the design of quantization algorithms, especially when embedded coding is required.

Images	Compression ratios	$PSNR_{dB}$			
		$FB_{REF}9/7$	FB9/7	FB13/11	FB17/11
Lena	16 :1	37,006	37,029	37,050	37,056
	32 :1	33,741	33,818	33,849	33,836
	64 :1	30,557	30,652	30,706	**30,713**
	128:1	27,589	**27,731**	**27,808**	27,825
Barbara	16 :1	31,199	31,343	**31,736**	31,705
	32 :1	27,438	27,530	27,815	**27,829**
	64 :1	24,554	24,620	24,760	**24,777**
	128:1	23,372	23,379	23,440	23,451
Fingerprint	16 :1	28,087	**28,330**	**28,586**	**28,597**
	32 :1	24,594	**24,775**	**25,103**	25,062
	64 :1	22,142	22,295	**22,597**	22,580
	128:1	19,541	19,744	**19,908**	19,883

Table 1. *PSNR (in dB) versus compression ratio for the three optimized filter banks and the reference filter bank* $FB_{REF}9/7$.

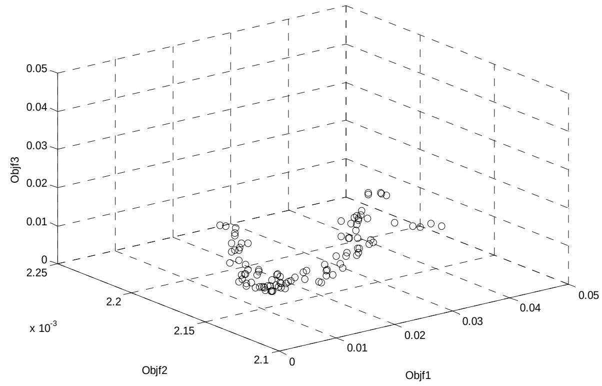

Figure 2. *3-D scatter plot of the Pareto-optimal solutions obtained by using the NSGAII (FB9/7)*

5 References

Deb K., A. Pratab, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," IEEE Trans. Evol. Comput., vol. 6, pp. 182–197, Apr. 2002.

Eskicioglu M. and Fisher P. S., "Image Quality Measures and Their Performance," *IEEE Transactions on Communications,* vol. 43, no. 12, pp. 2959-2965, 1995.

Horng B.R. and Willson A.N., "Lagrange multiplier approaches to the design of two-channel perfect-reconstruction linear-phase FIR filter banks," *IEEE Trans. Signal Process.,* vol 40, no.2, pp 364-374, 1992.

Katto J. and Yasuda Y., "Performance evaluation of subband coding and optimization of its filter coefficients," *Proc. SPIE Symposium on Visual Comm. and Image Proc.*, vol. 1605, pp.95-106, 1991.

Raghuwanshi M. M. and Kakde O. G., "Survey on multiobjective evolutionary and real coded genetic algorithms," *Proceedings of the 8th Asia Pacific Symposium on Intelligent and Evolutionary Systems*, pp. 150 – 161, 2004.

Said A. and Pearlman W.A., "A new, fast, efficient image codec based on set partitioning in hierarehical trees," IEEE Trans. Circuits Syst. Video Technol., vol.6, no.3, pp. 243-250, June 1996.

Shang Y. and Li L., "Optimization design of filter banks in subband image coding". Proceedings of the ASSET'99, pp.112 – 119, March 1999.

Villasenor J.D., Belzer B., and Liao J., "Wavelet filter evaluation for image compression," *IEEE Trans. Image Process.* vol.4, no.8, pp 1053-1060, 1995.

Vetterli M. and Kovačević J., "Wavelet and subband coding," Prentice Hall PTR, Englewood Cliffs, New Jersey, 1995.